# A New Ecological Model for Insect Population Size with Local Collapse

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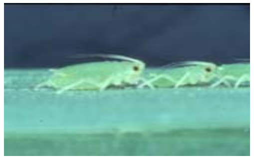
## Aphid Problem

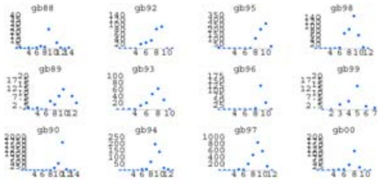
- Consider the Green Bug (GB)



GB is one of 4000 aphid species. Aphids are very destructive

## Data

- 12 years of GB counts, approx. weekly, at TAMU Bushland Station



- Each curve has local collapse. Max ranges from 20 to 2000.

We want population size model for GB (and other species)

## Logistic Model

- Let $N(t)$ = current count
- Assume $N'(t)=(r-sN)N$ (1)
- Alternative with carrying capacity $K=s/r$, $N'(t)=r\left(\frac{K-N}{N}\right)N$
- Solution:

$$N(t)=\frac{K}{1+\left[{(K-N_0)}/{N_0}\right]\exp(-rt)}$$

- Equilibrium solution is $K$

Logistic model doesn’t fit aphid data!

## Biological Principles of New Model

1. Aphids are parthenogenetic (virgin birth) and viviparous (live birth)

- Assume birth rate = $\lambda N$

2. Aphid population growth is constrained by the “cumulative size” of past population

- Let $F(t)=\int N(t)$=cumulative density

Assume death rate = $\delta F N$

New mechanistic model:

$$N'(t)=(\lambda-\delta F)N \qquad (2)$$

## Solutions

1. Prajneshu (JISAS 1998)

$$N(t)=a\exp(-bt)\left(1+d\exp(-bt)\right)^{-2}$$

$a$, $b$, $d$ are functions of $\lambda$, $\delta$, $N_o$

2. Our Alternative 1 (JABES 2006)

let $N_{\max}$= peak count, $t_{\max}$= time of peak

$$N(t)=\frac{4N_{\max}\exp\left(-b(t-t_{\max})\right)}{\left[1+\exp\left(-b(t-t_{\max})\right)\right]^2} \qquad (3)$$

3. Our Alternative 2

$$N(t)=N_{\max}\operatorname{sech}^2\left[{b(t-t_{\max})}/{2}\right] \qquad (4)$$

- Fit (3) or (4) to data. Could link to (2) using:

$$\lambda={b(d-1)}/{(d+1)}$$

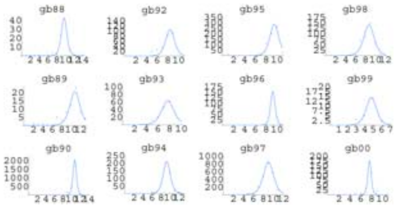
$$\delta={b^2}/{2N_{\max}}$$

$$N_o={4dN_{\max}}/{(1+d)^2}$$

where $d=\exp(bt_{\max})$

Model is symmetric with greater tails than the Normal

## Goodness-of-Fit for GB



Fit is near perfect

Claim 1: Model fits GB data very well. Same is true for pecan, mustard, cotton and corn leaf aphids

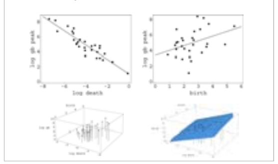
## Approach for Prediction

- Model predicts the full curve.
  Consider predicting only two endpoints:
  $y_1=O_{\max}$ (peak count) and $y_2=O_{\text{cum}}$ (cum density)
- Using full model, assume $y_i=f_i[\lambda,\delta,N(0)]+\varepsilon_i$,
  from whence $\log(y_i)=g_i[\lambda,\log(\delta),N(0)]+\varepsilon'_i$.
  Taylor series approx: $\log(y_i)=c_{0i}+c_{1i}\lambda+c_{2i}\log(\delta)+\varepsilon''_i$

## Linear Approximation

- How well does Taylor series model predict $O_{\max}$ and $O_{\text{cum}}$ ?

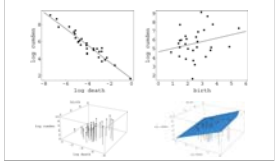
$O_{\max}$ for n=27 GB cases



- Multiple regression equation, with $R^2=0.98$:

$\log(O\text{max})=-0.48+0.60\lambda-0.48\log(\delta)$

$O_{\text{cum}}$ for n=27 GB cases



- Multiple regression equation, with $R^2=0.99$:

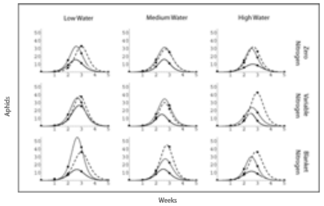
$\log(O\text{cum})=0.68+0.38\lambda-0.98\log(\delta)$,

Claim 2: Linear regression model, based on 1st order Taylor series approx, gives exceptional predictions of two endpoints, $O_{\max}$ and $O_{cum}$, based on mechanistic parameters $\lambda$ and $\delta$ for GB. Same is true for other aphids. This approach using $\lambda$ and $\delta$ is new and transparent.

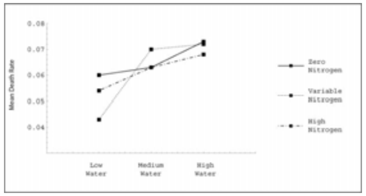
## Implementation

- Fact: $\lambda$ and $\delta$ explain almost all variability in $O_{\max}$ and $O_{\text{cum}}$ ($R^2 > 0.95$ in all such aphid studies)
- Implication: $\lambda$ and $\delta$ are implicit functions of local weather, predator, management, etc. conditions
- Our current research seeks to:

1. Relate $\lambda$ and $\delta$ to environmental variables.
   Example, consider cotton aphid abundance in 2 factor exp (water, nitrogen) in split-plot design in Texas in 2003.



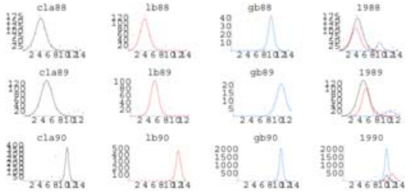
- Estimate $F(\infty)$, $t_{\max}$, $\delta$ for each curve.
- Finding: As water increases, $F(\infty)$ and $t_{\max}$, decrease, but $\delta$ increases.
- Profile plot of the mean $\delta$ vs. water:



- Note $\delta$ increased linearly with water in 2003 (a dry year)
- Also, $\lambda$ and $\delta$ decreased with nitrogen in 2004 (not a dry year)

2. Relate $\lambda$ and $\delta$ for GB to ladybug (predator) abundance.

- Plot of ladybug, GB, corn leaf aphid abundance for 3 years



Claim 3: Growth rate parameters $\lambda$ and $\delta$ are implicit functions of ambient conditions.

## Summary

1. New mechanistic model gives near-perfect fit for aphid data
2. New approach to prediction:

- observe local conditions
- postulate $\lambda$ and $\delta$
- predict $O_{\max}$ and $O_{\text{cum}}$

## Addendum

- Cotton aphid: another of the 4000 species